Math 253
Spring 2021

Name: Key

Quiz 5.5

1. (5 points) Does the Alternating Series Test apply to the series $\sum_{n=0}^{\infty} \frac{(-1)^n+1}{2^n}$? If yes, what conclusion does the Alternating Series Test allow you to make? If no, why not?

$$\sum_{n=0}^{\infty} \frac{(-1)^n+1}{2^n} = \frac{1+1}{1} + \frac{(-1)+1}{2} + \frac{1+1}{4} + \frac{(-1)+1}{8} + \cdots$$

$$= 2 + 0 + \frac{1}{2} + 0 + \cdots$$

The Alternating Series Test does not apply to $\sum_{n=0}^{\infty} \frac{(-1)^n+1}{2^n}$ because the terms are not alternating in sign.

2. (5 points) Does $\sum_{n=0}^{\infty} \frac{\cos(3n)}{n^2+1}$ converge absolutely, conditionally, or not at all?

$$0 \leq \left| \frac{\cos(3n)}{n^2+1} \right| \leq \frac{1}{n^2}$$

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by the p-test ($p = 2$),

so $\sum_{n=0}^{\infty} \left| \frac{\cos(3n)}{n^2+1} \right|$ converges by the Direct Comparison Test. Hence

$\sum_{n=0}^{\infty} \frac{\cos(3n)}{n^2+1}$ converges absolutely.